Labor Search

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For notes on sequential search, I refer you to the Acemoglu and Autor notes on labor Chapter 10, sections 1&2. I think the notes are excellent and extremely close to what we covered in lecture both in terms of content and notations.

Here I will write down the equilibrium search model we covered in class. Acemoglu and Autor notes chapter 11 does cover a similar model with capital investment. I encourage you to read that part too. I will present here a version without capital.

1 The environment

We consider an economy with workers and jobs. We are going to think of job and firms as conceptually the same thing through out. The mass of workers will be assumed to be one. The mass of firms is denoted N. Workers are either employed or unemployed. The mass of unemployed workers is denoted u. Jobs are either vacant or filled with a worker. We denote v the mass of vacancies and so the total number of jobs is given by $N = v + (1 - u)$.

Production and preferences

Unemployed workers enjoy a flow value of unemployment z. This captures potential government transfers as well as the difference in utility of leisure between working and not working. Vacant job pay a flow cost c which can be thought of as the cost to advertise the job to workers. When a job is filled with a worker, it generates output y. The firm can transfer money to the worker via the wage w. Both firms and workers care about discounted present value and discount at rate $\beta = \frac{1}{1+r}$. Filled jobs get separated at rate s which capture the risk that a given job is not viable anymore and needs to be terminated.

Matching technology

The meeting process between firms and workers is constrained by search frictions. Even when M workers seek a job and M vacancies are looking for workers only a share of these are able to find each other in a given period. This captures the fact that agent i) might not know where the jobs are located or ii) that two agents might apply to the same job creating coordination failure.

We define an object called the **tightness** by $\theta = \frac{v}{u}$. When it is hard for firms to hire workers, when θ is small, the labor market is described as slack. When it is easy to hire, it is described as tight, when the tightness is high. The **matching function** $m(u, v)$ is an element of the environment that gives the number of matches created when the equilibrium counts u unemployed workers and v vacancies. m describes the matching technology of the environment. You can think of it as a black box that tells you how strong are the frictions that prevent all vacancies and unemployed workers to meet each other. The matching function can be micro-founded. See Stevens (2002) for more details.

The probability for a vacancy to meet a worker is then $m(u, v)/v$ and the probability for a worker to meet a vacancy is $m(u, v)/u$.

In the case where the matching function is homogenous of degree 1, meaning $m(\lambda u, \lambda v) = \lambda m(u, v)$ we define the following functions

$$\begin{aligned} q(\theta) &= \frac{m(u,v)}{v} = m(\frac{u}{v}, 1) = m(\frac{1}{\theta}, 1) \\ p(\theta) &= \frac{m(u,v)}{u} = m(1, \theta) \end{aligned}$$

Values

Given the previous description, we can introduce definitions for present values for the different agents. We write V as the present value of a vacancy, J the present value of a job, E the present value of being employed and U the present value of being unemployed. We can express this values recursively:

$$\begin{aligned} J &= y - w + \beta\left[sV + (1-s)J\right] \\ V &= -c + \beta\left[q(\theta)J + (1-q(\theta))V\right] \\ E &= w + \beta\left[sU + (1-s)E\right] \\ U &= z + \beta\left[\theta q(\theta)E + (1-\theta q(\theta))U\right] \end{aligned}$$

This is true in a stationary environment. Consider the expression for the value of being employed E. When employed today, I start by collecting the wage w, then next period with probability s I become unemployed in which case I will get a present value of U. If I do not lose my job which happens with probability $1-s$ I remain employed and enjoy present value E by definition of E.

Nash bargaining

Because of the search frictions, there is a positive value associated with a filled job. This means that there are several possible wages that can make both the worker and the firm willing to be in a relationship versus being vacant and unemployed. Indeed the size of this surplus is given by

$$S = (J - V) + (E - U)$$

and when positive, we need to define how it is shared. We do so by imposing a Nash Bargaining condition. We assume that the surplus is shared according to

$$\begin{aligned} J - V &= (1-\phi)S \\ E - U &= \phi S \end{aligned}$$

where ϕ is the bargaining power of the worker.

Free entry condition

Finally we want to discipline the mass M of firms in the economy. We do so by imposing a free entry condition. We assume that

$$V = 0$$

Law of motion for unemployment

Given the known transition rates in and out of unemployment we can write a law of motion for unemployment:

$$u_{t+1} = s(1-u_t) + u_t\left(1 - \theta q(\theta)\right)$$

2 Equilibrium

Given an environment $m(\cdot,\cdot), y, z, \phi, \beta, s$ an equilibrium is a set of values J, V, E, U together with prices θ, w, firm mass M and unemployment u such that:

1. the value functions J, V, E, U solves the set of Bellman equations
2. w is such that the Nash bargaining condition is satisfied
3. the free entry condition is satisfied
4. θ, u and v are linked by the matching function
5. u satisfies the stationary unemployment law of motion
6. resource constraints are satisfied $N = v + (1-u)$

from all this conditions, we want to derive properties on the equilibrium of this model.

The Beveridge curve

The Beveridge curve is the law of motion for unemployment under the stationarity assumption $u_t = u_{t+1} = u$

$$u = \frac{s}{s + \theta q(\theta)} \tag{BC}$$

for different values of all parameters besides s and m, it locates the (u, v) point on a given curve. This has been documented as an empirical regularity.

The wage creation curve

We combine the J and V equations together with the free entry to get:

$$y - w = \frac{r+s}{q(\theta)}c \tag{JC}$$

this is like the demand curve in the friction-less labor market. Holding u constant, this equation links the vacancy creation (labor demand) as a function of the wage. This is a downward sloping curve. In appendix we see how it is derived from the J and V Bellman equations combined with the free entry condition

The wage curve

The wage curve represents the labor supply side of the market. Because there is no choice of hours in this very simple model it is a bit different. The curve comes out of the equations E and U and the wage bargaining equation:

$$w = z + \phi(y - z + \theta c) \tag{WC1}$$

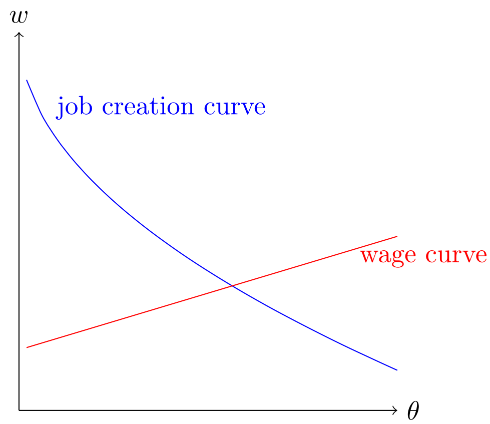
where one can replace further the vacancy cost c using the V equation to get the following second expression:

$$w = z + (y - z)\Gamma(\theta) \text{ where } \Gamma(\theta) = \frac{\phi(r + s + \theta q(\theta))}{r + s + \phi\theta q(\theta)} \tag{WC2}$$

This equation links the wage to the tightness θ, the bargaining power ϕ, the flow value of unemployment z the separation rate s and the interest rate r. The function $\Gamma(\theta)$ is increasing in θ. This equation gives us the wage w what the worker will be demanding in equilibrium via Nash Bargaining at given tightness. As the tightness increases, workers can find jobs faster, which increases their outside option U which, in turn, make them get higher wages in the bargaining phase. This curves reflects the fact that when the market is tight, workers can extract higher wages everything else equal.

Equilibrium tightness, unemployment and vacancies

By combining WC1 and JC we get an equilibrium definition for θ, this is the analog of the intersection of the supply and demand curve. Here we are in the (θ, w) space where in classical we were in the (L, w) space. Very much like in the efficiency wage exercise, here we choose a share of the economy that should work full-time, instead of choosing hours for everyone. The level of unemployment here is constrained by the matching function, which limits the amount of new matches as a function of θ.



using parameters:
r=0.1, s=0.1, c=0.1, y=1, ν=0.5, ϕ=0.3

To get u and v we combine the equilibrium θ with the Beveridge curve. Because is also true that $\theta = v/u$ at equilibrium we see that the locus of point in the Beveride curve diagram drawn by $u = \theta^* v$ is a straight line going through the origin.

And we can also now think in terms of comparative static how changing each of the parameters of the environment affects the endogenous components of the model. We get the following table:

	z	ϕ	h	m	y	s	r
w	+	+	-	+	+	-	-
θ	-	-	-	+	+	-	-
u	+	+	+	-	-	+	+

References

Stevens, M. (2002): "New microfoundations for the aggregate matching function, with empirical and theoretical implications," .

A Derivations

A.1 Derivation of the job creation curve

Using the condition that $V = 0$ we get:0

$$\begin{aligned} J &= y - w + \beta(1-s)J \\ 0 &= -c + \beta q(\theta) J \end{aligned}$$

where we can solve for J and get the job creation curve.

A.2 Derivation of the wage curve

we use the E and U equations:

$$\begin{aligned} E &= w + \beta\left[sU + (1-s)E\right] \\ U &= z + \beta\left[\theta q(\theta)E + (1-\theta q(\theta))U\right] \end{aligned}$$

we start by using the U equation to get $\beta\theta q(\theta)\,(E-U) = (1-\beta)U - z$ and then using the fact that $(1-\phi)\,(E-U) = \phi J$ from the bargaining equation. Then using $J = c/\beta q(\theta)$ we get

$$(1-\beta)U - z = \frac{\phi}{1-\phi}\frac{c}{\beta q(\theta)}$$

then from J and E we can write:

$$\begin{aligned} E &= \frac{w + \beta s U}{1-\beta(1-s)} \\ J &= \frac{y - w}{1-\beta(1-s)} \end{aligned}$$

which we can replace inside the equation $E - U = \phi S = \phi(E - U + J)$ to get

$$\frac{w + \beta s U}{1-\beta(1-s)} - U = \phi\left(\frac{y + \beta s U}{1-\beta(1-s)} - U\right)$$

which simplifies to

$$w - (1-\beta)U = \phi\left(y - (1-\beta)U\right)$$

where we can replace $(1-\beta)U$ with our previous expression $(1-\beta)U - z = \frac{\phi}{1-\phi}\frac{c}{\beta q(\theta)}$ to get

$$w = z + \phi(y - z + \theta c)$$