Income Shocks and Consumption Responses

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Abstract

This section talks about partial insurance, and how income shocks affect consumption decisions.

- Guvenen & Smith
- Blundell, Pistaferri, and Preston (2008)
  - evidence of transmission
- Heathcote, Storesletten, and Violante (2014)
  - deriving the solution to the problem
- Arellano, Blundell, and Bonhomme (2014)
  - identification of Mixture, HMM
  - introduce the homework
- Attanasio and Pavoni (2011)
  - excess sensitivity
- Guvenen, Karahan, Ozkan, and Song (2013), stanford guy paper on using Hours, Lise and Graber (2016) paper on using mobility
- Hours & job mobility

1 Income and consumption inequality

2 An analytical framework with close form solution

Here we study the paper by Heathcote, Storesletten, and Violante (2014). They develop a perpetual youth model, constant survival probability δ. The economy is composed of an infinite number of individuals in an infinite number of islands. Preferences over consumption and hours are given by:

$$\mathbb{E}_b \sum_{t=b}^{\infty} (\beta\delta)^{t-b} \, u(c_t(s^t), h_t(s^t); \phi) = \mathbb{E}_b \sum_{t=b}^{\infty} (\beta\delta)^{t-b} \left(\frac{c_t(s^t)^{1-\gamma}}{1-\gamma} - \exp(\varphi) \frac{h_t(s^t)^{1+\sigma}}{1+\sigma} \right)$$

and cohort born at time t draws $\phi_t \sim F_{\phi t}$. The structure of the shock is given by:

$$\begin{aligned} \log w_{it} &= \alpha_t^p + \epsilon_{it} \\ \alpha_t^p &= \alpha_{t-1}^p + \omega_t \\ \epsilon_{it} &= \kappa_{it}^p + \theta_{it} \\ \kappa_{it}^p &= \kappa_{it-1}^p + \eta_{it} \end{aligned}$$

2.1 Planner's problem

We start by solving the within period, within cohort, within Island allocation problem. The planner's problem at $x_{it} = (a_i, \varphi_i, \alpha_t)$ is to choose $c_t(x_{it}, \epsilon_{it})$ and $h_t(x_{it}, \epsilon_{it})$ as a solution of the following maximization problem:

$$\max_{\{c_t(x_t,\cdot),h_t(x_t,\cdot)\}} \int \left[\frac{c_t(x_{it}, \epsilon_{it})^{1-\gamma}}{1-\gamma} - \exp(\varphi_i) \frac{h_t(x_{it}, \epsilon_{it})^{1+\sigma}}{1+\sigma} \right] dF^a_{\epsilon,t}$$
$$\text{s.t.} \quad \int \left[\exp(\alpha_t + \epsilon_{it}) h_t(x_{it}, \epsilon_{it}) - c_t(x_{it}, \epsilon_{it}) \right] dF^a_{\epsilon,t}(\epsilon_{it}) = 0$$

call $\chi_t(x_t)$ the multiplier, the FOCs conditions are:

$$c_t(x_{it}, \epsilon_{it})^{-\gamma} = \chi_{it}(x_{it})$$
$$\exp(\varphi) h_t(x_t, \epsilon_t)^{\sigma} = \chi_t(x_t) \exp(\alpha_t + \epsilon_t)$$

we can note that the shock ϵ_{it} is fully insured in the consumption function. Combining the 2 equations gives:

$$h_t(x_t, \epsilon_t) = c_t(x_t)^{-\frac{\gamma}{\sigma}} \exp\left(\frac{\alpha_t + \epsilon_t - \varphi}{\sigma} \right)$$

which we then combine withe the resource constraint to get:

$$c_t(x_{it})^{-\frac{\gamma}{\sigma}} \exp\left(\frac{(1+\sigma)\alpha^p_t - \varphi}{\sigma} \right) \int \exp\left(\frac{(1+\sigma)\epsilon_{it}}{\sigma} \right) dF^a_{\epsilon,t} = c_t(x_{it})$$

or

$$\begin{aligned}
\log c_t(x_{it}) &= \frac{\sigma}{\sigma+\gamma} \left[\frac{(1+\sigma)\alpha^p_t - \varphi}{\sigma} + \log \int \exp\left(\frac{(1+\sigma)\epsilon_{it}}{\sigma} \right) dF^a_{\epsilon,t} \right] \\
&= \frac{1}{\sigma+\gamma} \left[(1+\sigma)\alpha^p_t - \varphi + \sigma \log \int \exp\left(\frac{(1+\sigma)\epsilon_{it}}{\sigma} \right) dF^a_{\epsilon,t} \right] \\
&= -\frac{\varphi}{\sigma+\gamma} + \frac{(1+\sigma)}{\sigma+\gamma}\alpha^p_t + \frac{\sigma}{\sigma+\gamma} \underbrace{\log \int \exp\left(\frac{(1+\sigma)\epsilon_{it}}{\sigma} \right) dF^a_{\epsilon,t}}_{\mathcal{C}^a_{it}} \\
\log c_t(x_{it}) &= -\frac{\varphi}{\sigma+\gamma} + \frac{1+\sigma}{\sigma+\gamma}\alpha_t + \frac{\sigma}{\sigma+\gamma}\mathcal{C}^a_{it}
\end{aligned}$$

next, we can get the same for hours

$$\begin{aligned}
\log h_t(x_t) &= -\frac{\gamma}{\sigma} \left(-\frac{\phi}{\sigma+\gamma} + \frac{(1+\sigma)}{\sigma+\gamma}\alpha_t + \frac{1+\sigma}{\sigma(\sigma+\gamma)}\mathcal{C}^a_{it} \right) + \frac{\alpha_t + \epsilon_t - \varphi}{\sigma} \\
\log h_t(x_t) &= \frac{1}{\sigma+\gamma}\varphi + \frac{1-\gamma}{\sigma+\gamma}\alpha_t + \frac{1}{\sigma}\epsilon_t - \frac{\gamma(1+\sigma)}{\sigma+\gamma}\mathcal{C}^a_{it}
\end{aligned}$$

2.2 Decentralization

We start by guessing the wage to be $w_t(x_t, \epsilon_t) = \exp(\alpha_t + \epsilon_t)$. The agent then solves the following problem:

$$\mathbb{E}_b \sum_{t=b}^{\infty} (\beta\delta)^{t-b} u(c_t, h_t; \phi)$$

with the following budget constraint:

$$
\begin{aligned}
w_t(s^t)h_t(s^t) + \delta^{-1}\left[B_{t-1}(s_t;s^{t-1}) + B^*_{t-1}(s_t;s^{t-1})\right] &= c_t(s^t) \\
&+ \int_S Q_t(s_{t+1};s^t)B_t(s_{t+1};s^t)ds_{t+1} \\
&+ \int_Z Q^*_t(s_{t+1};s^t)B^*_t(s_{t+1};s^t)dz_{t+1}
\end{aligned}
$$

We can write the full Lagrangian:

$$
\begin{aligned}
&\sum_{t=b}^{\infty}\int_{s^t}(\beta\delta)^{t-b}u(c_t(s^t),h_t(s^t);\phi)f_t(s^t)ds^t \\
-&\sum_{t=b}^{\infty}\int_{s_t}(\beta\delta)^{t-b}\lambda(s^t)\left(w_t(s^t)h_t(s^t)+\delta^{-1}\left[B_{t-1}(s_t;s^{t-1})+B^*_{t-1}(s_t;s^{t-1})\right]\right)f_t(s^t)ds^t
\end{aligned}
$$

where the FOCs give the following expressions, similar to the one we derived previously:

$$
\begin{aligned}
c_t(x_t,\epsilon_t)^{-\gamma} &= \lambda_t(s^t) \\
\exp(\varphi)h_t(x_t,\epsilon_t)^{\sigma} &= \lambda_t(s^t)\exp(\alpha_t+\epsilon_t)
\end{aligned}
$$

hence we get the same consumption rule. We can then use the FOC on $B_t(s_{t+1};s^t)$ to get that

$$
\begin{aligned}
\lambda_t(x_t)Q_t(S;s^t) &= \beta\delta\delta^{-1}\int_{s'\in S}\lambda_{t+1}(s':s^{t+1})f_{t+1}(s':s^t)/f_t(s^t)ds' \\
c_t(s^t)Q_t(S;s^t) &= \beta\delta\delta^{-1}\int_{s'\in S}c_{t+1}(s':s^{t+1})f_{t+1}(s'|s^t)ds'
\end{aligned}
$$

we then replace with the expression for consumption to get

$$
\begin{aligned}
Q_t(S;s^t) &= \beta\int_{s'\in S}\exp\left(-\gamma(\mathcal{C}^a_{i,t+1}-\mathcal{C}^a_{i,t})-\gamma\frac{1+\sigma}{\sigma+\gamma}\Delta\alpha^p_{t+1}\right)f_{t+1}(s'|s^t)ds' \\
&= \beta\exp\left(-\gamma(\mathcal{C}^a_{i,t+1}-\mathcal{C}^a_{i,t})\right)\int_{\omega_t\in S}\exp\left(-\gamma\frac{1+\sigma}{\sigma+\gamma}\omega_{t+1}\right)f_{\omega,t+1}(\omega_{t+1})d\omega_{t+1}Pr[\eta_{t+1},\theta_{t+1}]
\end{aligned}
$$

we then want to look at $\mathcal{C}^a_{t+1}-\mathcal{C}^a_t$ which gives (note that $\epsilon_{t+1}=\epsilon_t+\eta_{t+1}+\theta_{t+1}-\theta_t$):

$$
\mathcal{C}^a_{i,t+1}-\mathcal{C}^a_{i,t} = \frac{\sigma}{\sigma+\gamma}\log\left(\frac{\int\exp\left(\frac{(1+\sigma)}{\sigma}\eta_{it+1}\right)dF_{\eta,t+1}\int\exp\left(\frac{(1+\sigma)}{\sigma}\theta_{i,t+1}\right)dF_{\theta,t+1}}{\int\exp\left(\frac{(1+\sigma)}{\sigma}\theta_{i,t}\right)dF_{\theta,t}}\right)
$$

where we note that

- $\mathcal{C}^a_{t+1}-\mathcal{C}^a_t$ is not a function of s^t or age a.
- $\mathcal{C}^a_{t+1}-\mathcal{C}^a_t$ not a function of S either.

All consumption uncertainty is driven by the ω_t shock. We have that $Q_t(S;s^t)=Q_t(S)$ and that all pricing of the claim goes through the conditioning on ω_{t+1} and the probability of ϵ_{t+1}.

Next we look at the pricing of the claim across Island, and we use the no arbitrage rule to get that

$$\begin{aligned}
c_t(s^t)Q_t(S) &= \beta \int_{s' \in S} c_{t+1}(s' : s^{t+1}) f_{t+1}(s'|s^t) ds' \\
&= \int_{s' \in S} c_t(s^t) Q_t(s') f_{t+1}(s') ds' \\
Q_t(S) &= \int_{s' \in S} Q_t(s') f_{t+1}(s') ds'
\end{aligned}$$

Trade of the of insurance claims contingent on (η, θ). Agents could trade any contingency, or just a risk free bond. We first note that intra island trade is also available and so the inter island price is pinned down via the intra-island price and so:

$$Q^*(Z; s^t) = Q(Z; s^t) = Q(Z) = Q^*(Z) = Pr[(\eta, \theta) \in Z] \cdot Q(\mathbb{S})$$

We are considering an equilibrium without inter-island trade with prices driven by the guess we put forward. Under this pricing rule would there be gain from trade between agents on different Islands? Consider individuals with histories s^t_{i1} and s^t_{i2}, and consider

$$Q^*(Z; s^t) = Pr[(\eta_{t+1}, \theta_{t+1}) \in Z] \cdot Q_t(\mathbb{S})$$

as long as $Pr[(\eta_{t+1}, \theta_{t+1}) \in Z]$ and $Q_t(\mathbb{S})$ are the same across island, there is no gain from trade.

The final step is to derive the traded amount and verify that the individual budget constraint is satisfied.

3 Non parametric earnings and consumption

In this section we are going to follow Arellano, Blundell, and Bonhomme (2014) which develops the identification and the estimation of a non linear earning function as well as non parametric consumption functions. The earnings process is decomposed as follows:

$$\begin{aligned}
&y_{it} = \eta_{it} + \epsilon_{it} \\
&\eta_{it} = Q_t(\eta_{i,t=1}, u_{it}),\ (u_{it}|\eta_{i,t-1}, \eta_{i,t-2}..) \sim Uniform(0,1)
\end{aligned}$$

where we see that a particular case is given by $\eta_{it} = \eta_{i,t-1} + v_{it}$. Secondly we the paper consider a general consumption rule of the form

$$c_{it} = g_t(a_{it}, \eta_{it}, \epsilon_{it}, \nu_{it})$$

3.1 On the identification of the earnings process

To simplify the exposition here, we are going to consider the case where variables are discrete. We are going to proceed in multiple steps. We first look at the identification of finite mixture, we then extend this and look at the identification of hidden Markov chain in panel data. In both case we are interested in estimating a model of the joint distribution of earnings $(y_{it}, y_{i,t+1}, y_{i,t+2}...)$. In both cases, we extend the model with a latent variable η_{it}. For simplicity of exposure, in this section of the course we will focus on discrete latent variables and often discretize outcomes as well.

3.1.1 NP identification of discrete mixtures

We first consider a model with the following assumptions on (y_{it}, η_{it}):

$$\begin{aligned}
&\eta_{it} = \eta_i \\
&y_{it} \perp y_{it'} | \eta_i
\end{aligned}$$

under such conditions, consider three realizations of the wage:

$$\begin{aligned} Pr[Y_1 \leq y_1, Y_2 \leq y_2, Y_3 \leq y_3] &= \sum_k Pr[\eta_k] Pr[Y_1|\eta_k] Pr[Y_2|Y_1, \eta_k] Pr[Y_3|Y_1, Y_2, \eta_k] \\ &= \sum_k Pr[\eta_k] Pr[Y_1|\eta_k] Pr[Y_2, \eta_k] Pr[Y_3|\eta_k] \end{aligned}$$

consider then, a discretized version of the outcome. We pick a set of n wage levels $(y_1, ...y_n)$ generating n bins. We then construct the following matrices:

$$\begin{aligned} [A(y_3)]_{pq} &= Pr[Y_1 \leq y_p, Y_2 \leq y_q, Y_3 \leq y_3] \\ [F]_{pq} &= Pr[Y \leq y_p|\eta_q] \\ [D(y_3)]_{pq} &= \delta_{pq} Pr[\eta_p] Pr[Y \leq y_3|\eta_p] \end{aligned}$$

We can then rewrite our previous expression as:

$$\begin{aligned} [A(y_3)]_{pq} &= \sum_k [D(y_3)]_{kk} [F]_{pk} [F]_{kq} \\ A(y_3) &= F D(y_3) F^\intercal \end{aligned}$$

then we consider the svd decomposition of $A(y_3) = USV^t$ and note that we have that:

$$\begin{aligned} I_d &= S^{-\frac{1}{2}} U^\intercal A(y_3) V S^{-\frac{1}{2}} \\ &= \underbrace{S^{-\frac{1}{2}} U^\intercal F D(y_3)}_{Q} \underbrace{F^\intercal V S^{-\frac{1}{2}}}_{Q^{-1}} \end{aligned}$$

compute

$$\begin{aligned} S^{-\frac{1}{2}} U^\intercal A(y_3') V S^{-\frac{1}{2}} &= S^{-\frac{1}{2}} U^\intercal A(y_3') V S^{-\frac{1}{2}} \\ &= S^{-\frac{1}{2}} U^\intercal F D(y_3') F^\intercal V S^{-\frac{1}{2}} \\ &= \underbrace{S^{-\frac{1}{2}} U^\intercal F D(y_3)}_{Q} D(y_3)^{-1} D(y_3') \underbrace{F^\intercal V S^{-\frac{1}{2}}}_{Q^{-1}} \\ &= Q D(y_3)^{-1} D(y_3') Q^{-1} \end{aligned}$$

we we see can be recovered by Eigen value decomposition. The requirements for the uniqueness of the decomposition are that all the eigen values are different. Here this means that the diagonal elements of $D(y_3)^{-1} D(y_3')$ are all distinct. This returns Q and $D(y_3)^{-1} D(y_3')$. Finally note that

$$F^\intercal V S^{-\frac{1}{2}} = Q^{-1}$$

and so

$$F = Q^{-1} S^{\frac{1}{2}} V^\intercal$$

which proves the identification result (there is a normalization to be done, we use the fact that $F(\infty) = 1$.

3.1.2 NP identification of hidden Markov chain

Here we follow Hu and Shum (2012). We consider the following restrictions on the joint (y_{it}, η_{it}) process:

$$\begin{aligned} Pr[Y_t|\eta_t, \Omega_t] &= Pr[Y_t|\eta_t] \\ Pr[\eta_t|\eta_{t-t}, \Omega_{t-1}] &= Pr[\eta_t|\eta_{t-t}] \end{aligned}$$

We can show here again that with only 3 observation we can recover the emission distribution as well as a the transition rule. We start with the same joint probability as what we did in the in previous section:

$$
\begin{aligned}
Pr[Y_1 \le y_1, Y_2 \le y_2, Y_3 \le y_3] &= \sum_{k_1}\sum_{k_2} Pr[Y_1, Y_2, Y_3, \eta_{k_1}, \eta_{k_2}] \\
&= \sum_{k_1}\sum_{k_2} Pr[Y_3|\eta_{k_2}]Pr[Y_2, \eta_{k_2}|Y_1, \eta_{k_1}]Pr[Y_1, \eta_{k_1}] \\
&= \sum_{k_1}\sum_{k_2} Pr[Y_3|Y_2, \eta_{k_2}]Pr[Y_2|\eta_{k_2}]Pr[\eta_{k_2}|\eta_{k_1}]Pr[Y_1, \eta_{k_1}] \\
&= \sum_{k_2} Pr[Y_3|Y_2, \eta_{k_2}]Pr[Y_2|\eta_{k_2}] \left(\sum_{k_1} Pr[\eta_{k_2}|\eta_{k_1}]Pr[Y_1, \eta_{k_1}]\right) \\
&= \sum_{k_2} Pr[Y_3|Y_2, \eta_{k_2}]Pr[Y_2|\eta_{k_2}] \left(\sum_{k_1} Pr[\eta_{k_2}|\eta_{k_1}]Pr[Y_1|\eta_{k_1}]Pr[\eta_{k_1}]\right) \\
&= \sum_{k_2} Pr[Y_3|\eta_{k_2}]Pr[Y_2|\eta_{k_2}]Pr[\eta_{k_2}, Y_1]
\end{aligned}
$$

We use the same $A(y_3)$ as before, here we use $N = K$ for simplicity. We introduce the following additional matrices:

$$
\begin{aligned}
[F_{Y_2 Y_1}]_{pq} &= Pr[Y_2 \le y_p, Y_1 \le y_q] \\
[F_{Y_1,\eta_2}]_{pl} &= Pr[Y_1 \le y_p, \eta_2 = \eta_l] \\
[D(y_3)]_{kl} &= \delta_{k=l} Pr[Y_3 \le y_3|\eta_2 = \eta_k] \\
[G]_{kl} &= Pr[\eta_2 = \eta_k, \eta_1 = \eta_l]
\end{aligned}
$$

then we note that

$$
\begin{aligned}
Pr[Y_2, Y_1] &= \sum_{k_2} Pr[Y_2|\eta_{k_2}, Y_1]Pr[Y_1, \eta_{k_2}] \\
&= \sum_{k_2} Pr[Y_2|\eta_{k_2}]Pr[Y_1, \eta_{k_2}]
\end{aligned}
$$

and we have in matrix notations:

$$
\begin{aligned}
[F_{Y_2 Y_1}]_{pq} &= \sum_{k_2} \left[F_{Y_2|\eta_2}\right]_{pk_2} [F_{Y_1,\eta_2}]_{k_2 q} \\
F_{Y_2 Y_1} &= F_{Y_2|\eta_2} F^{\intercal}_{Y_1,\eta_2}
\end{aligned}
$$

and from the previous expression we had that

$$
\begin{aligned}
[A(y_3)]_{pq} &= \sum_{k_2} [D(y_3)]_{k_2 k_2} [F_{Y_1,\eta_2}]_{k_2 q} [F_{Y_1,\eta_2}]_{pk_2} \\
A(y_3) &= F_{Y_2|\eta_2} D(y_3) F^{\intercal}_{Y_1,\eta_2}
\end{aligned}
$$

we can then compute

$$
A(y_3) F^{-1}_{Y_2 Y_1} = F_{Y_2|\eta_2} D(y_3) F^{-1}_{Y_2|\eta_2}
$$

which by eigen value decomposition gives us directly the emission probability F. The condition for identification here is that the diagonal element of D are all distinct or that the following are distinct:$Pr[Y \le y_3|\eta_{k_2}]$.

Finally, we want to recover the law of motion. For this we decompose again $Pr[Y_2, Y_1]$.

$$
\begin{aligned}
Pr[Y_2, Y_1] &= \sum_{k_2} \sum_{k_1} Pr[Y_2|\eta_{k_2}] Pr[\eta_{k_2}, \eta_{k_1}] Pr[Y_1|\eta_{k_1}] \\
[B]_{pq} &= \sum_{k_2} \sum_{k_1} [F]_{pk_2} [G]_{k_2 k_1} [F]_{k_2 q} \\
B &= F G F^{\intercal}
\end{aligned}
$$

where we see that we can recover $G = F^{-1} B \left(F^{\intercal}\right)^{-1}$

3.2 Estimation using EM

3.2.1 General Formulation of EM

3.2.2 Gaussian mixture

Let's run through using the EM algorithm in details for the finite Mixture model, considering a normal mixture model. We write the probability model as:

$$
L(Y_1, Y_2, Y_3; \theta) = Pr[Y_1{=}y_1, Y_2{=}y_2, Y_3{=}y_3; \theta] = \sum_{k=1}^{K} p_k \prod_{t=1}^{3} \phi(Y_t; \mu_k, \sigma_k)
$$

where the parameter space is given by $\theta = \{p_k, \mu_k, \sigma_k\}_{k=1..K}$. The Maximum Likelihood estimator solves:

$$
\theta^{MLE} = \arg\max_{\theta} \sum_i \log L(Y_{i1}, Y_{i2}, Y_{i3}; \theta)
$$

The EM algorithm is an iterative procedure that climbs the likelihood surface. Given a parameter guess $\theta = \{p_k, \mu_k, \sigma_k\}_{k=1..K}$, the algorithm is composed of two steps:

In the Expectation step we compute:

$$
q_i(k) = Pr[\eta_i = \eta_k | Y_{i1}, Y_{i2}, Y_{i3}; \theta^{\tau}]
$$

a natural procedure is to compute this posterior probability using the likelihood model:

$$
\begin{aligned}
q_i(k) &= \frac{Pr[\eta_i = \eta_k | Y_{i1}, Y_{i2}, Y_{i3}; \theta^{\tau}]}{\sum_l Pr[\eta_i = \eta_l | Y_{i1}, Y_{i2}, Y_{i3}; \theta^{\tau}]} \\
&= Pr[\eta_i = \eta_k | Y_{i1}, Y_{i2}, Y_{i3}; \theta^{\tau}] \\
&= \frac{p_k^{\tau} \prod_{t=1}^{3} \phi(Y_t; \mu_k^{\tau}, \sigma_k^{\tau})}{\sum_{l=1}^{K} p_l^{\tau} \prod_{t=1}^{3} \phi(Y_t; \mu_l^{\tau}, \sigma_l^{\tau})}
\end{aligned}
$$

The second step uses these probabilities in the maximization step:

$$
\max_{\theta} \sum_i \sum_k q_i(k) \log Pr(Y_1, Y_2, Y_3, \eta_k; \theta)
$$

where in our case

$$
Pr(Y_1, Y_2, Y_3, \eta_k; \theta) = p_k \prod_{t=1}^{3} \phi(Y_t; \mu_k, \sigma_k)
$$

where we get

$$
\max_{\theta} \sum_i \sum_k q_i(k) \left(\log p_k^{\tau+1} - \log \sqrt{\pi} \sigma_k^{\tau+1} - \sum_t \frac{\left(Y_{it} - \mu_k^{\tau+1}\right)^2}{2\left(\sigma_k^{\tau+1}\right)^2} \right)
$$

the FOC of p_k gives

$$p_k^{\tau+1} = \frac{\sum q_i(k)}{N}$$

and the FOCs for μ_k gives

$$\sum_i \sum_t q_i(k) \left(Y_{it} - \mu_k\right) = 0$$

or in other in other words:

$$\mu_k^{\tau} = \frac{\sum_i q_i(k) \sum_t Y_{it}}{T \cdot \sum_i q_i(k)}$$

and finally the estimate of the variance is given by

$$\sigma_k^{\tau+1} = \sqrt{\frac{\sum_i \sum_t q_i(k) \left(Y_{it} - \mu_k^{\tau+1}\right)^2}{T \cdot \sum_i q_i(k)}}$$

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