Classical Labor Supply: Micro and Macro Elasticities

ECON 34430: Topics in Labor Markets

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Recap on Micro Vs Macro

- Intensive Frisch elasticity is low in many estimated micro studies, which suggests that all intensive elasticities are low
  - see review surveys (Keane: two groups, Saez: basicaly 0, Chetty:)
- Macro Representative agent seems to require high Frisch elasticity of aggregate hours
  - Prescott (2004) uses around 2.0 to explain cross-country analysis
- reconciliations ?
  - Keane suggests that intensive micro is actually not so low (include HC)
  - Heckman, Chetty point to differences between extensive and intensive elasticities
  - Rogerson Wallenius show that micro and macro might not be directly linked
  - Chetty points out that disagreement mostly in Frisch

Today: labor supplys aggregation

- Chang and Kim (2006)
  - develops a Macro model with worker heterogeneity
  - matches Business cycle facts
  - compares estimated micro/macro elasticities

- Attanasio and Al (2015)
  - consider an integrated model of intratemporal and intertemporal labor supply choices at both the intensive and the extensive margins.
  - consider aggregation problem directly

Prescott 2004 - Why Do Americans Work So Much More Than Europeans?

Intro - rules of the game

1. write a simple macro model of labor supply and taxation
2. calibrate the model with identical parameters for all countries
3. apply countries specific tax codes
4. how much does this explain of labor supply differences?

Output, Labor Supply, and Productivity

In Selected Countries in 1993–96 and 1970–74

Period	Country	Relative to United States (U.S. = 100)		
		Output per Person*	Hours Worked per Person*	Output per Hour Worked
1993–96	Germany	74	75	99
	France	74	68	110
	Italy	57	64	90
	Canada	79	88	89
	United Kingdom	67	88	76
	Japan	78	104	74
	United States	100	100	100
1970–74	Germany	75	105	72
	France	77	105	74
	Italy	53	82	65
	Canada	86	94	91
	United Kingdom	68	110	62
	Japan	62	127	49
	United States	100	100	100

*These data are for persons aged 15–64.

The model 1

- stand-in household with preference

$$E[\sum_{t=0}^{\infty} \beta^t \left(\log c_t + \alpha \log(100 - h_t) \right)] \quad (1)$$

- law of motion for capital stock

$$k_{t+1} = (1 - \delta)k_t + x_t$$

- stand-in firm with market clearing

$$y_t = c_t + x_t + g_t \leq A_{it} k_t^{\theta} h_t^{1-\theta}$$

where g_t is public spending

The model 2

- the budget constraint for the household is given by:

$$
\begin{aligned}
&(1-\tau_c)c_t + (1+\tau_x)x_t \\
&\qquad = (1-\tau_h)w_t h_t + (1-\tau_k)(r_t-\delta)k_t + \delta k_t + T_t
\end{aligned}
$$

- r_t is rental price of capital
- $\tau_x, \tau_c, \tau_h, \tau_k$ are taxes on consumption, investment, labor and capital income and define $\tau = (\tau_h + \tau_c)/(1+\tau_c)$
- T_t is a lump sum transfer

Equilibrium relationships

- marginal rate of substitution between leisure and consumption

$$\frac{\alpha/(1-h)}{1/c} = (1-\tau)w$$

- wage and marginal product of labor

$$w = (1-\theta)k^{\theta}h^{-\theta}$$

- which we combine to get

$$h_{it} = \frac{1-\theta}{1-\theta+\frac{c_{it}}{y_{it}}\frac{\alpha}{1-\tau_{it}}}$$

Equilibrium relationships

- the following expression captures most of the trade-offs:

$$h_{it} = \frac{1-\theta}{1-\theta+\frac{c_{it}}{y_{it}}\frac{\alpha}{1-\tau_{it}}}$$

- $1-\tau$ affects the relative price of between consumption in leisure within a period

- c/y which is directly impacting x and as such the capital stock, reflects the inter-temporal decision

- bottom line is that this expression links h to c, y, α, τ

Estimating tax rates 1

- define Indirect Tax on consumption as a function of total IT and C, I:

$$IT_c = (\underbrace{2/3}_{\text{priv cons exp}} + 1/3 \cdot \frac{C}{C+I})IT$$

- this captures that most IT falls on consumption (value added, sales) but some falls on capital investment (sales tax on equipment, property tax on office building)

- and consumption and output as

$$c = C + G - G_{mil} - IT_c$$
$$y = GRP - IT$$

where G is public consumption, G_{mil} is military

Estimating tax rates 2

- consumption tax rate is given by

$$\tau_c = \frac{IT_c}{C - IT_c}$$

- value for the social security tax is

$$\tau_{ss} = \frac{\text{Social Security Taxes}}{(1-\theta)(GDP - IT)}$$

where the denominator is labor income when labor is paid marginal product

Estimating tax rates 3

- the average income tax is given by:

$$\bar{\tau}_{inc} = \frac{\text{Direct Taxes}}{GDP - IT - \text{Depreciation}}$$

- the marginal income tax is set to:

$$\tau_h = \tau_{ss} + 1.6\bar{\tau}_{inc}$$

- finally we need to parametrize as follows, from what I understood:
  - $\theta = 0.32$ using wage equation?
  - $\alpha = 1.54$ to match the average value of h ?

Labor supply, actual and predicted

Period	Country	Labor Supply*		Differences (Predicted Less Actual)	Prediction Factors	
		Actual	Predicted		Tax Rate τ	Consumption/ Output (c/y)
1993–96	Germany	19.3	19.5	.2	.59	.74
	France	17.5	19.5	2.0	.59	.74
	Italy	16.5	18.8	2.3	.64	.69
	Canada	22.9	21.3	−1.6	.52	.77
	United Kingdom	22.8	22.8	0	.44	.83
	Japan	27.0	29.0	2.0	.37	.68
	United States	25.9	24.6	−1.3	.40	.81
1970–74	Germany	24.6	24.6	0	.52	.66
	France	24.4	25.4	1.0	.49	.66
	Italy	19.2	28.3	9.1	.41	.66
	Canada	22.2	25.6	3.4	.44	.72
	United Kingdom	25.9	24.0	−1.9	.45	.77
	Japan	29.8	35.8	6.0	.25	.60
	United States	23.5	26.4	2.9	.40	.74

Overview of results 1/2

1. surprisingly close! given everything else is ignored

2. in Germany, France and Italy, low participation is explained by high taxes
   - when European and US tax rate were similar, labor supply was comparable
   - US vs France/Germany differences can be explained by differences in tax rates

Overview of results 2/2

1. a second interesting point is the evolution of labor supply in the US
   - despite tax rates remaining similar, participation went up
   - Prescott argues that marginal tax rates of moving from one wage earner to 2 in household was much lower in 93-96 and in 70-74. And that increased participation was mostly among married women.

2. counterfactual calculations give:
   - in France, reducing from 0.6 to 0.4 lifetime consumption would go up by 19%
   - in the US, reducing from 0.4 to 0.3 lifetime consumption would go up by 7%

What is the value of the labor elasticity?

- Chetty, uses participation data and the numbers from the paper to get both extensive and intensive margin from this model
- He reports 0.25 for the Hicksian extensive and around 0.33 for the intensive (this is in the high end of the micro values)
- I took the values from the table and regressed log hours on log net-of-tax rate and found 0.69981 for the elasticity of aggregate hour.
- Chetty reports an average extensive of 0.25 and Keane gives an average Hicksian of 0.31 which would give around 0.56 total response. This is not so far.

Lower α

- What if I pick α to match the micro Hicksian elasticity of 0.56?
- Using R I found that it requires $\alpha = 0.55$ instead of 1.54
- this might generate much weaker responses to taxes
- The main disagreement according to Chetty is on the Frisch extensive elasticity

RogersonWallenius 2008 - Micro and macro elasticities in life cycle model with taxes

Intro

- the paper argues that one problem is to not consider separaty extensive and intensive margins
- it builds on Prescott and introduces both a choice on amount of time per period and share of life spent working
- using the model they compare predicted micro and macro elasticities
- and they look at the effect of changing marginal tax rate

The model 1

- continuous time overlapping generation model
- life of an individual is normalized to 1
- at each instant t, individual are endowed with 1 unit of time
- denote by a the age of the agent, preferences are:

$$\int_0^1 U(c(a), 1 - h(a))da$$

- agents choose consumption and work hours paths $c(a), h(a)$
- no discounting, zero interest rate steady state
- The government taxes labor income at rate τ and redistributes it as a uniform lump-sum.

The model 2

- labor is the only factor of production, output is given by $Y(t) = L(t)$.

- $L(t)$ is the input of labor services

- agents hours is mapped into labor services according to:

$$l = e(a) \cdot g(h)$$

- $e(a)$ captures variation in life cycle productivity
  - provides a driving force for life-cycle employment decision
  - assumed to be single peaked

- $g(h)$ captures potential fixed cost of working
  - $g(h) = \max 0, h - \bar{h}$
  - the convexity implies that it could be optimal to randomize some agents to work full time and other to not work.
  - hourly wage rate for part time will be lower

Equilibrium 1

- time zero markets for labor and consumption $w(t), p(t)$
- market are competitive, production is linear so $w(t) = p(t)$
- the presence of markets allows for agent to implicitly trade between period at interest rate $p(t)/p(t')$
- the authors focus on zero interest rate steady state equlibrium, $p(t)$ is constant, so is $w(t)$. They can be both normalized to 1

Equilibrium 2

- a new born optimization problem is given by:

$$\max_{c(a),h(a)} \int_0^1 U(c(a), 1-h(a)da$$
$$\text{s.t.} \quad \int_0^1 c(a)da = \int_0^1 e(a)g(h(a))da$$

- first consider $e(a)$ to be constant
  - case 1: $h(a) > 0 \forall a$ then h is constant
  - case 2: $h(a) > 0$ only in some places then only fraction is pinned down, not locations of hours worked
  - this could be the case to deal with convexities in g

General case

1. the paper shows that $h^*(a)$ has a reservation property:

$$\exists e^* : h^*(a) > 0 \Leftrightarrow e(a) > e^*$$

   - this removes the indeterminacy of the location of work over the life-cycle
   - the assumption that $e(a)$ is single peaked will mean that there will be a unique starting and stopping age for working

2. the paper also shows that for amount of hours worked we have that

$$e(a_1) \geq e(a_2) \Rightarrow h^*(a_1) \geq h^*(a_2)$$

3. both property will generate life-cycle participation and hours

Calibration

- for the quantitative section, the model is calibrated in the following way:
- $U(C, 1-h) = \log(c) - \alpha \frac{h^{1+\gamma}}{1+\gamma}$
- $g(h) = \max\{0, h - \bar{h}\}$
- $e(a) = e_0 - e_1|.5 - a|$
- for different values of γ, pick $\alpha, \bar{h}, e_1$ to match:
  - λ fraction of life spent unemployed
  - h_{max} peak hour of work over the life cycle
  - variation in hourly earnings over the life cycle
- model is calibrated with tax of 0.3

Matching wages in the data and in the model

- remember $w(t) = 1$!
- wage is earnings per hour of work
- if g was linear then we would get $e(.5)/e(a^{max})$
- define $w^h(a) = e(a)g(h(a))/h(a)$
- then the targeted wage ratio is

$$\frac{w^h(.5)}{w^h(a^{max})} = \frac{e(.5)g(h(.5))/h(.5)}{e(a^{max})g(h(a^{max}))/h(a^{max})}$$

- choose e_1 to get a wage ratio of 2

Generated micro elasticities

- the author generate data from the calibrated model
- they then run the following regression:

$$\log(h_t) = b_0 + b_1 \log(w_t^h) + \epsilon_t$$

- this should reflect the estimated micro Hicks elasticity

Estimated values of b_1.

$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 10$
1.29	.59	.28	.05

- the non-linearity of g implies that the measured elasticity is very different from $1/\gamma$ (remember here $\eta = 0$)

Changing the tax transfer

- using calibrated model, tax is changed from 0.3 to 0.5

Relative outcomes for $\tau = .5$.

γ	H	λ	$h^{\max}$
.50	.777	.857	.856
1.00	.784	.825	.918
2.00	.788	.808	.956
10.00	.790	.794	.991

- H is aggregate hours, λ is fraction of life being employed, h^{max} is peak hour

1. aggreate hours goes down 20%
2. the change in aggregate hours is unaffected by changes in the γ parameters, and as such by the estimated micro-elasticities
3. shift in γ affects the break down of the change in total hours between λ and h^{max}

Conclusion

- a simple model with extensive and intensive margins
- no clean link between γ and estimated micro elasticity
- no effect of γ or estimated micro-elasticities on how taxes affect aggregate hours
- yet this model would have serious problem replicating cross-country numbers put forward by Prescott

References