Sorting in the labor Market

Part 2: Theory of sorting

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Introduction to Part 2

Develop the theory to understand:

- How is sorting linked to fundamentals like production?
- How does sorting arise in equilibrium?
- How are wages set, impacted by sorting, and linked to productivity?

In this section we will go over:

1. static frictionless matching: Becker (1974)
2. introducing random search: Shimer and Smith (2000)
3. going to the data: Hagedorn, Law, and Manovskii (2014)

Frictionless matching

Frictionless matching with TU

Environment:

- fixed measure of workers indexed by $x \in \mathbb{X}$ (uniform)
- fixed measure of jobs indexed by $y \in \mathbb{Y}$ (uniform)
- production function $f(x, y)$
- assume common ranking $f_x > 0, f_y > 0$
- ability for matched agents to transfer to each other w (wage)

Preferences:

- firms care about profits : $\pi = f(x, y) - w$
- workers care about wages: w

Allocation defined by a matching rule (μ, w):

- $\mu(x) = y$: who matches with whom (assuming pure for today)
- $w(x)$: a wage schedule

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Frictionless matching with TU: equilibrium

Stable matching rule:

- No pair (x, y) can do better than in equilibrium

$$\forall x, y : \quad \underbrace{w(x)}_{x \text{ eq pay off}} + \underbrace{\pi(\mu^{-1}(y), y)}_{y \text{ eq pay off}} \geq \underbrace{f(x, y)}_{\text{potential output}}$$

Results:

- Existence: **YES** (Shapley and Shubik, 1971)
- Efficiency: **YES** Maximizes joint utility
- Unique: Matching is generically unique, transfers are not
- Stable Eq and Competitive Eq coincide (Gretsky, Ostroy, and Zame, 1999)

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Competitive Eq and Assortative matching

- Firm's problem
  - Takes wage schedule as given and chooses x to max profit
  $$\max_x f(x,y) - w(x)$$
  - FOC: $f_x(x,y) - w_x(x) = 0$
  - SOC: $f_{xx}(x,y) - \underbrace{w_{xx}(x)}_{?} < 0$
- Eq condition at FOC
$$\forall x \qquad f_x(x,\mu(x)) - w_x(x) = 0$$
$$f_{xx}(x,\mu(x)) + f_{xy}(x,\mu(x))\mu'(x) - w_{xx}(x) = 0$$
- Assortative matching relationship:
$$f_{xy}(x,\mu(x)) \cdot \mu'(x) > 0$$

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Production function and assortative matching

- PAM (NAM) is optimal when $f_{xy} > 0$ ($f_{xy} < 0$)
  - If production is super-modular, better workers in better firms is more efficient
  - Gives positive implication for assignment of workers to firms
- Super-modularity is about the change in the change:
  - Do better worker gain more from moving to better firms
- Note for empirical work:
  - When matching is pure, can't differentiate worker from firm effect
  - Difficult to generate wage dispersion for similar workers or mismatch

Matching with search frictions

Matching with frictions: environment 1/2

Environment:

- fixed measure of workers indexed by $x \in \mathbb{X}$ (uniform)
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- production function $f(x, y)$
- assume common ranking $f_x > 0, f_y > 0$
- ability for matched agents to transfer to each other w (wage)
- unemployed workers get $b(x)$; vacancies pay $c(y)$
- worker and firms care about EPV (forward looking)

Allocation:

- $u(x)$ mass of unemployed workers, $v(x)$ mass of vacancies
- $h(x, y)$ mass of matches (like μ, but not pure anymore)
- $w(x, y)$ wage and $M(x, y)$ matching decision

Matching with frictions: environment 2/2

Matching process:

- the meeting between unemployed workers and vacancies is constrained by search frictions
  - unemployed workers find offers at rate λ
  - vacancies find workers at rate μ
  - λ and μ can be endogenized with a matching function ▷
- matching is random
  - workers sample from $v(y)$, firms sample from $u(x)$

Timing:

1. production: matches collect output and pay the wage
2. meeting: unemployed workers and vacancy meet
3. matching: newly matched pairs decide whether to start a partnership ($M(x, y)$)
4. separation: existing matches separate at exogenous rate δ

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Matching with frictions: match surplus

Present values:

- workers and firms are forward looking
- $W_1(x, y, w)$ and $W_0(x)$ EPV for employed and unemployed
- $\Pi_1(x, y, w)$ and $\Pi_0(y)$ EPV for job and vacancy
- the surplus of a match is define as:

$$S(x, y) := W_1(x, y, w) + \Pi_1(x, y, w) - W_0(x) - \Pi_0(y)$$

Value of the surplus details ▷

$$(r + \delta)S(x, y) = (1 + r)f(x, y) - rW_0(x) - r\Pi_0(y)$$

- Given TU, matching decision is $M(x, y) = 1[S(x, y) \geq 0]$
- The surplus can be non-monotonic because of option value
- Surplus complmentarity directly inherited from $f(x, y)$

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Wages and division of surplus:

- A continuum of way to split the surplus!
- Additional assumption that surplus is split via Nash bargaining
- define α as the worker bargaining power then $w(x, y)$ solves:

$$(1-\alpha)\Big(W_1(x,y,w) - W_0(x)\Big) = \alpha\Big(\Pi_1(x,y,w) - \Pi_0(y)\Big)$$

- which implies that upon meeting
  - worker gets $W_0(x) + \alpha S(x, y)$
  - firm gets $\Pi_0(x) + (1-\alpha) S(x, y)$

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Final elements

EPV unemployed

$$rW_0(x) = (1+r)b(x) + \lambda \int \alpha M(x,y) S(x,y) \frac{v(y)}{V} \,\mathrm{d}y$$

EPV vacancy

$$r\Pi_0(y) = -(1+r)c(y) + \mu \int (1-\alpha) M(x,y) S(x,y) \frac{u(x)}{U} \,\mathrm{d}x$$

Matching distribution

$$\delta \cdot h(x,y) = \frac{\lambda}{V} M(x,y) u(x) v(y)$$

Equilibrium

Given the primitives $f(x,y), c(y), b(x), r, \delta, \alpha, \lambda, \mu$ a **stationary search equilibrium** is

- defined by:
  - EPVs: $S(x,y)$, $\Pi_0(y)$, $W_0(x)$, $\Pi_1(x,y,w)$, $W_1(x,y,w)$
  - allocations $h(x,y), u(x), v(y)$
  - wage $w(x,y)$ and matching decision $M(x,y)$
- such that:
  1. the value functions solve the Bellman equations
  2. the wage is the Nash bargaining solution
  3. the distribution satify the stationary equation and adding up

Some results

- Existence: YES (Shimer and Smith, 2000)
- Uniqueness: NO
- Efficiency: not in general
  - workers do not internalize how they affect others search
  - room for efficiency improving policies
- Assortative matching
  - Shimer and Smith (2000) introduces new definition: monotonicity of matching set boundaries
  - log supermodular $f(x, y) \implies$ PAM
  - log submodular $f(x, y) \implies$ NAM
  - requires stronger complementarity than in friction-less

See it in action

- parametrize production function

$$f(x, y) = (ax^{\rho} + (1 - a)y^{\rho})^{1/\rho}$$

- consider PAM and NAM
- allocation, wages, mean wages
- live demo $\implies$ let's start R!

- parametrize production function

$$f(x, y) = (ax^{\rho} + (1 - a)y^{\rho})^{1/\rho}$$

- consider PAM and NAM
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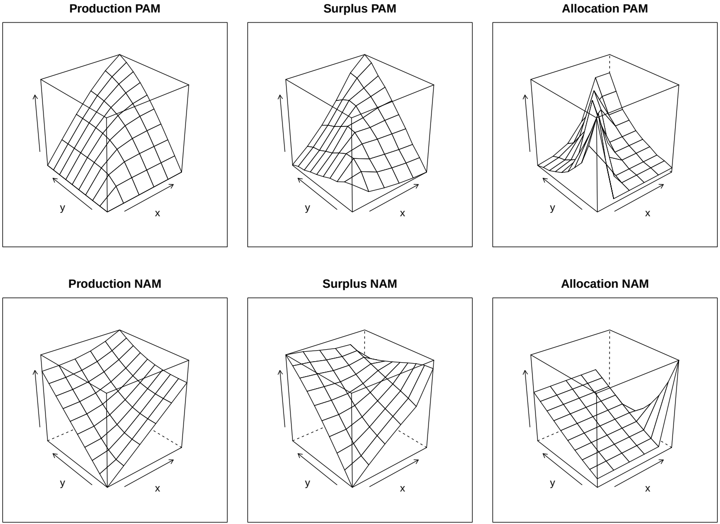
Matching with frictions and AKM specification

- We have a model that generates:
  - sorting in equilibrium
  - wage dispersion within firm
  - mobility from firms to firms (via unemployment)
- However, the wage does not comply with AKM specification
  - the wage is not log-linear additive in general
  - more importantly, the wage does not seem monotonic in y!
- What happens if we simulate from Shimer-Smith and run AKM?

Theoretical search-matching model: plots



Notes: Model based on Shimer and Smith (2000).

HLM: simulate from SS and estimate AKM

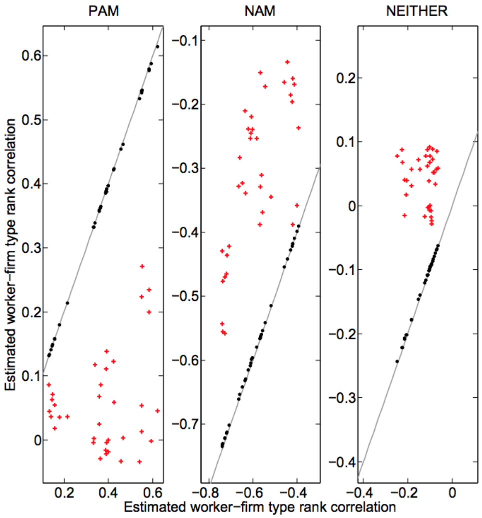
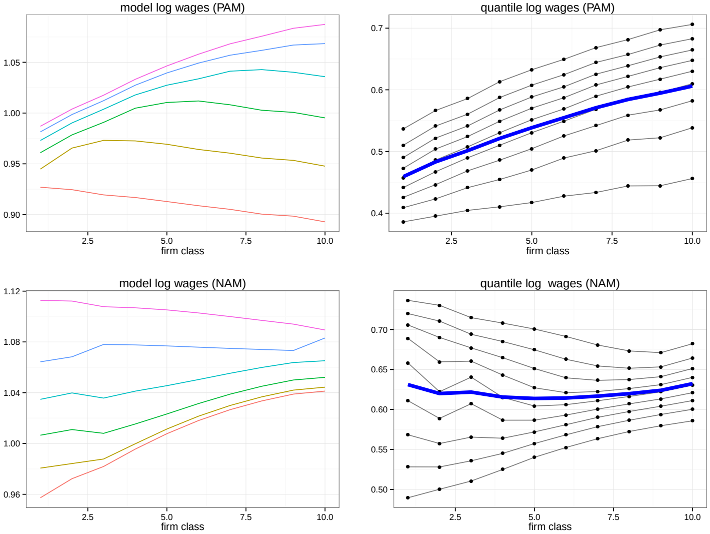


Figure 3: Correlation between identified worker and firm ranks against true correlation.

Theoretical search-matching model: wage distributions



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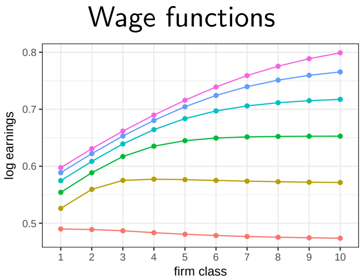
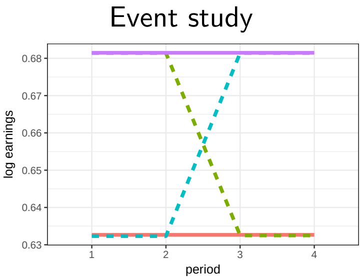




Identification and estimation of the search model

Estimating the structural search model

- Hagedorn, Law, and Manovskii (2014) proposes a constructive way of estimating the search model

  1. rank workers using monotonicity of wages within firm
  2. construct a monotonic measure of firm type
  3. once type are observed everything follows

Step 1: Rank workers

- remember the equilibrium wage:

$$(1+r)w(x,y) + \delta\, W_0(x) = (r+\delta)\Big(W_0(x) + \alpha S(x,y)\Big)$$

which gives:

$$(1+r)w(x,y) = r(1-\alpha)\,W_0(x) + \alpha(1+r)f(x,y) - r\alpha\Pi_0(y)$$

- $w(x,y) \nearrow$ in x, so within a firm y we can rank workers
- with movers, we can compare workers accross firms
- with enough movers a general rank can be aggregated
- the paper develops a rank aggregation algorithm

Step 1: Rank firms

- we start from the value of a vacancy which is increasing in y

$$r\Pi_0(y) = -(1+r)c(y) + \mu \int (1-\alpha) M(x,y) S(x,y) \frac{u(x)}{U} \,\mathrm{d}x$$

- we then use the expression for the wage of the worker

$$(1+r)w(x,y) = rW_0(x) + \alpha(r+\delta)S(x,y)$$

- where if $S = 0$ we get the lowest accepted wage for type x $(1+r)\underline{\mathrm{w}}(x) = rW_0(x)$

- replacing gives us an observable monotonic measure of firm types:

$$\Omega(y) = \int M(x,y)\Big(w(x,y) - \underline{\mathrm{w}}(x)\Big)\frac{u(x)}{U} \,\mathrm{d}x$$

HLM performance

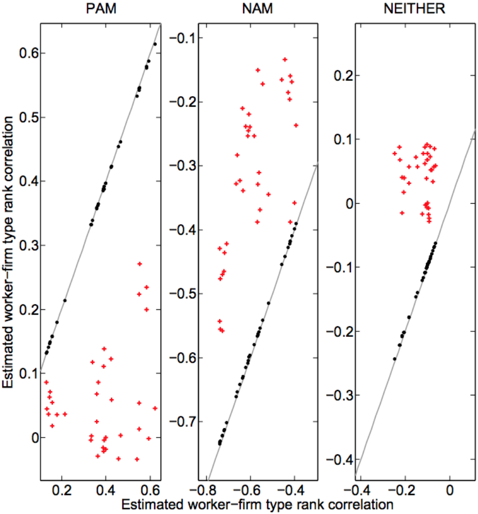


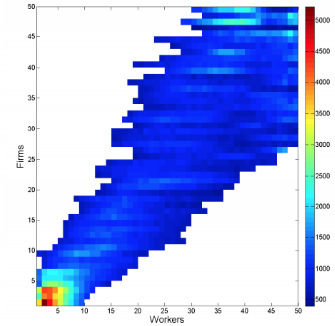
Figure 3: Correlation between identified worker and firm ranks against true correlation.

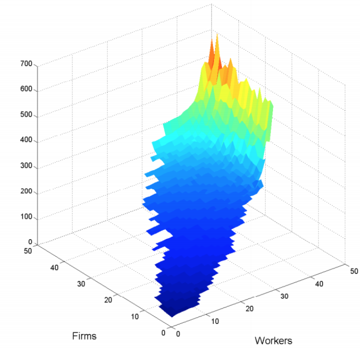
HLM on german data

- Estimate on the same data as Card, Heining, and Kline (2013)
- Reports very strong sorting and complementarities

Table 3: Sign and Strength of Sorting

	HLM	AKM
Corr(W-rank,F-rank)	0.7132	0.055





Limitations

- wage measurement error affects rank property within firm
- the method does not generalize to OTJ (2/3 of job moves)
  - wages grow $\implies$ w/r is not the present value anymore
  - wages are history dependent $\implies$ the ranking within firm is lost
  - the surplus equation is not directly linked to production
  - one needs to work with EPV of wages (Lamadon, Lise, Meghir, and Robin, 2013)
- Eeckhout and Kircher (2011) shows that when discounting is close to 1, it is difficult to get the sign of sorting. It applies to this procedure. There is a race between discounting and strength of complementarity.
- no inference

Conclusion

- we looked at the main theory of sorting
- with complementarity in production, better workers in better firms is more efficient
- these models of sorting seems incompatible with the AKM empirical model (both in principle and in practice)
- Going further:
  - Bagger and Lentz (2014); Lamadon, Lise, Meghir, and Robin (2013) develop identification and estimate models with OTJ
  - Abowd, Kramarz, Pérez-Duarte, and Schmutte (2009) develop a model with restricted production where wages are log-linear additive and there is sorting in equilibrium (no OTJ)
- Can we develop an empirical method compatible with Becker type sorting? This is the next section!

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Surplus derivation

- $(r+\delta)\,W_1(x,y,w) = (1+r)w + \delta\,W_0(x)$
- $(r+\delta)\Pi_1(x,y,w) = (1+r)(f(x,y) - w) + \delta\Pi_0(y)$
- by definition of the surplus:

$$(r+\delta)S(x,y) = (1+r)f(x,y) - rW_0(x) - r\Pi_0(y)$$

back ▷

Matching function

- number of matches is given by $N = m(U, V)$
- then $\lambda = \frac{N}{U}$ and $\mu = \frac{N}{V}$
- a classic matching function is $m(u, v) = au^{0.5}v^{0.5}$

References

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